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Eberlein-compact 空間とその仲間

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Eberlein-compact 空間の研究の流れから Talagrand-compact, Gul'ko-compact, Corson-compact 等の空間が出てきた。これらの空間の特徴付けを整理しながら未解決の問題を紹介する。

以下空間は Tychonoff とする。空間 X 上の実数値連続関数全体を C(X) で表す。C(X) 上に各点収束位相を導入した空間を $C_p(X)$ で表す。

定義。空間 X をコンパクト空間とする。

空間 X があるバナッハ空間 E の弱コンパクト部分空間と位相同型となるとき、この空間 X を Eberlein-compact (略して、E-C) と呼ぶ [AL]。

空間 X に対して、Cp(X) が $\mathcal{K}$-analytic となる時、即ち Cp(X) が $K_{\sigma\delta}$-空間の連続像となる時、空間 X を Talagrand-compact (T-C) と呼ぶ。

空間 X に対して、Cp(X) が Lindelof Σ-空間となる時、空間 X を Gul'ko-compact (G-C) と呼ぶ。

空間 X が Σ-product of real lines の部分空間と位相同型となる時、空間 X を Corson-compact (C-C) と呼ぶ。

$$E-C \;\underset{\not\Leftarrow\,[N]}{\overset{[T1]}{\Rightarrow}}\; T-C \;\underset{\not\Leftarrow\,[T3]}{\overset{[T2]}{\Rightarrow}}\; G-C \;\underset{\not\Leftarrow\,[N]}{\overset{[Gu]}{\Rightarrow}}\; C-C$$

特徴付けを表にすると以下のようになる。

『?』の記号が付いている所は未解決である。

X compact space	E-C	T-C	G-C	C-C
Cp(X) P:irrationals K:compact $P' \subset P$	cont.image of a sep.metric space × compact space. [A]	cont.image of a closed subspace $F \subset P \times K$.	cont.image of a closed subspace $F \subset P' \times K$	cont.image of a closed subspace $F \subset L(T)^{\omega}$ [P]
$X \subset \Sigma(T)$ $\Sigma(T)$:Σ-product of real lines	$X \subset \Sigma(T)$:Σ -product [AL]	$X \subset \Sigma(T)$ such that (a) [S]	$X \subset \Sigma(T)$ such that (b) [S]	$X \subset \Sigma(T)$:Σ-product
T_0-separating coll. of open F_σ-sets	σ-point-finite [R]	(a') [S]	weakly σ-point finite [S]	point-countable [MR]
X^2	hereditarily σ-metacompat [G1]	?	hereditarily weakly σ-metacompact ? [G3]	hereditarily metaLindelof [G1]
$X^2-\Delta$	σ-metacompact [G1]	?	weakly σ-metacompact ? [G3]	metaLindelof [G1]
game $G(\Delta, X^2)$	O has Markov wining strategy [G2]	?	?	W-set [G1]

①. The space L(T) is a union of a discrete space T and a point at infinity ∞ whose nbds are of the form $\{\infty\} \cup (T-C)$, where Cis countable.
②. (a),(a') and (b) ;see Sokolov [S] Theorem 9, Theorem 8.

③. A collection $\mathcal{U}$ of subsets of X is weakly σ-point-finite if $\mathcal{U}=\cup\{\mathcal{U}_n : n\in\omega\}$ so that, for each $x\in X$, $\mathcal{U}=\cup\{\mathcal{U}_n : \mathrm{ord}(x,\mathcal{U}_n)<\omega\}$.

④. A space X is weakly σ-metacompact if every open cover has a weakly σ-point-finite open refinement.

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